Modeling the p-dispersion problem with distance constraints*

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Abstract

We study the p-dispersion problem with distance constraints (pDD), a variant of the well-known p-dispersion problem. In a pDD, the goal is to locate a set of facilities so as to maximize the minimum distance between any two of them, subject to additional constraints specifying minimum allowed distances. Two CP models for the pDD have recently been proposed. The first is a typical model that includes the global constraints *Minimum* and *Element* and explicitly represents the objective function, connecting it to the decision variables. However, as problem size grows, this model becomes increasingly inefficient. The second model adopts a simplistic approach that only uses binary constraints, essentially treating the pDD as a satisfaction problem. In this paper, after demonstrating the deficiencies of these models, we propose a new compact model that captures the problem through ternary constraints, instead of global or binary ones. We prove that, rather surprisingly, the pruning of the decision variables' domains achieved in our new model is equivalent to that achieved in the model with global constraints, resulting in the same search tree under the same variable and value ordering. Experiments demonstrate that our new model is by far superior to the existing ones, both in terms of solution quality and run times.

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1 Extended Abstract

Maximum diversity problems arise in many practical settings, from facility location to telecommunications [4, 2]. The most famous is the maxmin p-dispersion problem [15, 6, 14], which is $\mathcal{NP}$-hard in general networks. Given a set of candidate locations $P = \{1, 2, \ldots, n\}$ for p facilities and a distance matrix $D[l][m]$, the goal is to locate the p facilities to maximize the minimum distance between any pair. This is typically relevant when locating (ob)noxious facilities (e.g., power plants, dump sites) or when dispersing franchises to limit direct competition [4, 8].

The p-dispersion problem with distance constraints (pDD) is a variant where additional distance constraints exist between facilities [9, 3, 1, 17, 16]. The standard CP model can be formulated in any CP solver, allowing for the pDD to be solved to optimality through Branch&Bound search [12]. In this model, the decision variables correspond to the facilities and there are auxiliary variables representing the pairwise distances between facility points.

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These sets of variables are linked via `Element` constraints. The distance constraints are expressed as arithmetic relations imposed on the auxiliary variables. Alternatively, `Table` constraints can be used to link decision and auxiliary variables and to capture the distance constraints. A `Minimum` constraint is imposed on all auxiliary variables and the objective variable z, forcing the latter to equal the minimum pairwise distance between the selected locations. The objective is to maximize the value of z. Experiments have indicated that the size of the model grows rapidly with the number of facilities and potential location points, primarily due to the introduction of numerous auxiliary variables and constraints, often leading to memory exhaustion and system crashes [12].

For this reason, a much simpler CP model, using only p decision variables and binary distance constraints was proposed [12]. While it requires significantly less memory, it lacks an explicit objective function. As a result, improving solutions do not strongly propagate their cost, forcing the solver to essentially treat the pDD as a satisfaction problem and blindly enumerate non-improving solutions. Despite its simplicity and weaker propagation power, this model was shown to be necessary when dealing with instances with large numbers of facilities and candidate locations because of its low memory requirements. In the context of this model, a greedy branch-pruning heuristic was also proposed [5, 12]. The use of such a heuristic was shown to drastically reduce the search space; however, its application turns any CP solver from exact to inexact.

In this paper, we focus on the efficient exact solving of the pDD. Our initial experiments with the state-of-the-art solvers CP Optimizer [7], OscaR [10], Choco [13] and OR-Tools CP-SAT [11], confirmed that the models with global constraints suffer from excessive memory consumption and runtime overhead, while the binary model struggles with weak objective propagation, discovering a huge amount of non-improving solutions. Given the above, the challenge is to find a way to combine the strengths of both approaches, i.e., strong propagation with lightweight modeling, so that large problems of realistic size can be handled.

To address this, we propose a simple and efficient CP model for the pDD. The core idea is to introduce a variable that models the objective in the binary model and to include this variable in all distance constraints, turning them into ternary from binary. This establishes a direct link between the decisions and the objective without requiring any global constraints (like `Minimum`, `Table` or `Element`). Rather surprisingly, we show that although we maintain the compactness of the simple model with only p decision variables and $(p \times (p-1)/2)$ constraints, the pruning of the decision variables' domains achieved in our new model is equivalent to that achieved in the model with global constraints. Hence, under the same variable and value ordering, search explores the same search tree in both models.

We implemented all models (including the proposed one with its custom propagator) in the Choco solver and evaluated them on large, realistic instances (up to 2,000 locations and 200 facilities, including a waste-bin location case study). Using lexicographic variable and value ordering, results demonstrate that in our new model run times can be significantly shorter, allowing the solver to prove optimality in smaller classes within 1-hour of cpu time in many instances where search in the model with global constraints failed to terminate. Experiments with dynamic variable ordering, which allows for larger instances to be solved, demonstrate that our new model significantly reduces computation time, while yielding solutions of much higher quality compared to the other models and being able to handle very large instances that are beyond their reach. Ultimately, our proposed ternary model offers a highly efficient, exact, and scalable alternative for solving large-scale pDDs.

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