Cardinality Cuts And Saturation

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Abstract

We compare the complexity of a range of new cutting planes (CP) subsystems that exploit cardinality constraints. We show CP paired with cuts from linear programming and mixed integer programming can be powerful especially when considering such cardinality constraints. Through this investigation we re-examine the power of saturation and find an exponential refutation separation between it and these new subsystems.

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1 Introduction

Pseudo-Boolean (PB) constraints, which express integer linear inequalities over Boolean variables, offer a significantly more expressive and succinct framework than standard Conjunctive Normal Form (CNF) clauses. The natural proof system underlying conflict-driven PB solvers is Cutting Planes (CP), which traditionally generates derivations of new constraints through linear combinations and division (or saturation) rules. However, with the rise of PB solvers such as RoudingSAT [3] it is of particular interest to investigate restricted subsystems of cutting planes that can boost performance while remaining practically feasible.

To date, the proof complexity of these PB subsystems has primarily centered on the interplay between division and saturation. Vinyals et al. [6] proved that saturation-based systems with polynomially bounded coefficients are polynomially simulated by standard resolution. While they proposed candidate formulas suggesting division might be exponentially stronger than saturation, they were unable to formally establish this separation. To bridge this gap, Gocht et al. [5] formally separated the two rules. They established that division, even when heavily restricted to cancelling addition (generalized resolution), can be exponentially more powerful than saturation with unrestricted linear combinations. Conversely, they showed that simulating even a single saturation step can require a non constant number of division operations.

Operations research has long been interested in a range of separate derivation rules for knapsack like problems within Integer Programming. Such rules include cover cuts which derive clauses and extended cover cuts which derive cardinality constraints [1]. Cardinality constraints have seen practical implementations in pseudo boolean solvers [2] yet to the best of our knowledge have not been reviewed from a proof complexity perspective against current cutting planes rules. In this paper, we explore how the proof complexity of saturation and cover cut rules compare.

2 Preliminaries

By pseudo-Boolean (PB) constraints we always mean 0-1 integer linear constraints. In what follows, all such constraints are assumed to be written in normalized form as non-negative linear combinations of literals $\sum_i a_i l_i \geq A$, where the coefficients $a_i \in \mathbb{N}_0$ are non-negative integers, the degree of falsity $A \in \mathbb{N}_+$ is a positive integer, and literals l_i represent variables x_i or negated variables $\bar{x}_i$ (which cancel to produce $x_i + \bar{x}_i = 1$, and where at most one

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of x_i and $\bar{x}_i$ appears in any constraint). Constraints of the form $\sum_i a_i x_i \leq C$ are trivially transformed into normalized form by negating the variables.

The cutting planes proof system can be defined as consisting of rules for literal axioms

$$l_i \geq 0 \ , \tag{1}$$

linear combination

$$\frac{\sum_i a_i l_i \geq A \qquad \sum_i b_i l_i \geq B}{\sum_i (c_A a_i + c_B b_i) l_i \geq c_A A + c_B B} \qquad [c_A, c_B \in \mathbb{N}_0] \ , \tag{2}$$

and division

$$\frac{\sum_i a_i l_i \geq A}{\sum_i \lceil a_i / c \rceil l_i \geq \lceil A/c \rceil} \qquad [c \in \mathbb{N}_+] \ . \tag{3}$$

An alternative to using division is the saturation rule:

$$\frac{\sum_i a_i l_i \geq A}{\sum_i \min(a_i, A) l_i \geq A} \ . \tag{4}$$

We denote the proof system utilizing literal axioms, linear combinations, and saturation as CP+Sat.

In addition to the cutting planes rules above, there also exists the cover cut rule from integer programming for capacity constraints of the form $\sum_j a_j x_j \leq C$.

A subset of variables S is a *cover* if its total weight strictly exceeds the capacity, $\sum_{j \in S} a_j > C$. Because the variables in S cannot all simultaneously be assigned to 1, we can derive the basic cover cut:

$$\sum_{j \in S} x_j \leq |S| - 1 \ . \tag{5}$$

In CP normal form this gives us $\sum_{j \in S} \bar{x}_j \geq |S| - 1$. It can be observed that such a constraint will always output a clause. We shall refer to CP (without division or saturation) paired with this rule as CP+Cover.

Given a cover S, let $M = \max_{k \in S} a_k$ be the maximum weight in S and the *extended set* $E = \{j \notin S \mid a_j \geq M\}$. This yields the extended cover cut:

$$\frac{\sum_j a_j x_j \leq C}{\sum_{j \in S \cup E} x_j \leq |S| - 1} \ . \tag{6}$$

Expressed over negative literals, this cut takes the equivalent form $\sum_{j \in S \cup E} \bar{x}_j \geq |E| + 1$. We refer to constraints of this form as cardinality constraints.

3 The Separating Formula

▶ **Theorem 1** (Main Separation). *There exists a family of pseudo-Boolean formulas $\{F_n\}_{n=1}^{\infty}$ of size polynomial in n such that:*

1. *F_n has a polynomial-size refutation in CP+Sat.*
2. *Any refutation of F_n in CP+Cover requires size $2^{\Omega(n)}$.*

3.1 The Formula F_n

Let $n \in \mathbb{N}^+$. The formula F_n is a pseudo-Boolean instance over matrix variables $x_{ij} \in \{0,1\}$ for $i \in [4n+3]$, $j \in [2n+1]$ and auxiliary variables $y_{ik} \in \{0,1\}$ for $i \in [2n+1]$ and $k \in [n]$. For each $i \in [2n+1]$, we partition the $2n$ off-diagonal indices $[2n+1] \setminus \{i\}$ into two disjoint sets S_1^i and S_2^i such that $S_1^i \cup S_2^i = [2n+1] \setminus \{i\}$ and $|S_1^i| = |S_2^i| = n$.

▶ **Definition 2** (Formula F_n). *The formula F_n consists of the following sets of constraints:*

- ***Split rows*** *($i \in [2n+1]$):*

$$nx_{ii} + \sum_{k=1}^{n} y_{ik} + \sum_{j \in S_1^i} x_{ij} \geq n \tag{7}$$

$$nx_{ii} + \sum_{k=1}^{n} \bar{y}_{ik} + \sum_{j \in S_2^i} x_{ij} \geq n \tag{8}$$

- ***Lower rows*** *($i \in \{2n+2, \ldots, 4n+3\}$):*

$$\sum_{j=1}^{2n+1} x_{ij} \geq n \tag{9}$$

- ***Columns*** *($j \in [2n+1]$):*

$$nx_{jj} + \sum_{i \neq j} x_{ij} \leq 2n \tag{10}$$

By adding the two split row constraints (7) and (8) for a given $i \in [2n+1]$, the auxiliary variables cancel out ($\sum_{k=1}^{n} y_{ik} + \sum_{k=1}^{n} \bar{y}_{ik} = n$), yielding the *combined upper row*:

$$2nx_{ii} + \sum_{j \neq i} x_{ij} \geq n \tag{11}$$

In CP+Sat, any coefficient exceeding the right-hand threshold can be reduced instantly. Applying the saturation rule to (11) reduces the coefficient of x_{ii} from $2n$ to n, yielding the saturated constraint:

$$nx_{ii} + \sum_{j \neq i} x_{ij} \geq n \tag{12}$$

To build our intuition for the upper and lower bounds, it is helpful to view the matrix variables x_{ij} as representing a bipartite network flow problem. In this view, the row constraints represent "demand" that must be satisfied, and the column constraints represent the maximum "capacity" or "supply" available to satisfy that demand.

▶ **Lemma 3** (Upper Bound). *The formula F_n has a CP+Sat refutation of size $\mathcal{O}(n)$.*

Proof. For each $i \in [2n+1]$, summing the split row axioms (7) and (8) eliminates the auxiliary y-variables. Saturating the resulting inequality yields the saturated upper row (12). Summing these $2n+1$ saturated upper rows together with all $2n+2$ lower rows (9) yields a global variable demand of $4n^2+3n$. However, summing all $2n+1$ column capacity axioms (10) restricts this exact same set of variables to a maximum global supply of $4n^2+2n$. Resolving these two aggregated bounds yields the immediate contradiction $0 \geq n$. Since each row and column is processed a constant number of times, the total size of the refutation is $\mathcal{O}(n)$. ◀

4 The Exponential Lower Bound for CP+Cover

In this section, we prove the second half of Theorem 1: that CP+Cover requires exponential size to refute F_n. Our proof proceeds in two main phases. First, we establish a width lower bound, proving that no narrow clause can be derived even under a noisy partial assignment. Second, we apply a random restriction that eliminates wide clauses with high probability, forcing the proof system into the narrow-clause regime where it becomes permanently stuck.

▶ **Theorem 4** (Width Lower Bound). *Let A be any partial assignment to the variables of F_n fixing at most $n/16$ variables in any single row or column, and fixing at most $n/16$ diagonals x_{ii} in total. We define the x-*width *of a clause as the total number of primary matrix variables x_{ij} it contains. Then no clause D of x-width $\leq n/16$ over the A-free variables can be derived by CP+Cover from $F_n|_A$.*

Proof. Let D be an arbitrary clause of width $|D| \leq n/16$ over the variables left free by A. Let α be the unique minimal assignment to these free variables that falsifies D. To prove that D cannot be derived, we must demonstrate that $F_n|_{A\cup\alpha}$ is locally consistent. We establish this via the following lemma. ◀

▶ **Lemma 5** (Local Fractional Consistency). *There exists a real-valued extension $\beta \in [0, 1]$ for all remaining free variables such that β strictly satisfies all constraints in $F_n|_{A\cup\alpha}$.*

Proof. **4.0.0.1 Setup and Row Partitioning.**

We partition the upper rows $i \in [2n + 1]$ into three sets based on the status of their diagonal variables x_{ii} under $A \cup \alpha$: P_1 (fixed to 1), P_0 (fixed to 0), and P^* (completely unassigned). Let $|D_{\text{fixed}}| = |P_1| + |P_0|$ be the total number of fixed diagonals. Since A fixes at most $n/16$ diagonals by hypothesis, and α fixes at most $|D| \leq n/16$ variables in total, we have $|D_{\text{fixed}}| \leq n/16 + n/16 = n/8$.

For each upper row i, let k_i denote the number of off-diagonal variables x_{ij} $(j \neq i)$ fixed to 1 by $A \cup \alpha$. A fixes at most $n/16$ ones per row, and α contributes at most $|D| \leq n/16$ ones in total, ensuring $k_i \leq n/16 + n/16 = n/8$.

4.0.0.2 Fractional Assignment to the Upper Matrix.

We assign values $\beta(x_{ij})$ and $\beta(y_{ik})$ to the remaining free variables to explicitly satisfy the split row constraints (7) and (8). Because A leaves all auxiliary variables unassigned and α fixes at most $n/16$ variables in total, every row retains at least $15n/16$ free y_{ik} variables. This provides ample continuous slack to balance the split constraints via a fractional assignment $\beta(y_{ik}) \in [0, 1]$.

- **For $i \in P_1$:** The diagonal $x_{ii} = 1$. We set all free off-diagonals x_{ij} and all free auxiliary variables y_{ik} to 0. Because $x_{ii} = 1$, the split rows demand $n(1) + \cdots \geq n$, which are both strictly satisfied regardless of the auxiliary values.
- **For $i \in P_0$:** The diagonal $x_{ii} = 0$. We must fractionally assign exactly $n - k_i$ weight across the free off-diagonals. To do this constructively, we uniformly distribute this weight across the remaining free variables in S_1^i and S_2^i. Since $A \cup \alpha$ fixes at most $n/8$ variables per row, each partition retains at least $7n/8$ free variables. By setting each free x_{ij} to an equal fraction, the total weights W_1 and W_2 naturally balance such that the ample remaining free y_{ik} variables can easily be set fractionally to satisfy the exact inequalities of (7) and (8).

- **For $i \in P^*$:** We set the fractional diagonal $\beta(x_{ii}) = \frac{n-k_i}{2n} \in [0,1]$. No pumping of off-diagonals is needed; all remaining free off-diagonals in the row are set to 0.For split rows let $k_{i,1}$ and $k_{i,2}$ denote the number of off-diagonals fixed to 1 in S_1^i and S_2^i respectively (where $k_{i,1} + k_{i,2} = k_i$). Substituting $\beta(x_{ii})$ into the split rows reveals that they are both satisfied if and only if the total sum of the auxiliary variables is exactly:

$$\sum_{k=1}^{n} \beta(y_{ik}) = \frac{n + k_{i,2} - k_{i,1}}{2} \ .$$

  Because the noise guarantees $k_{i,1}, k_{i,2} \leq n/8$, this required target sum falls in the range $[\frac{7}{16}n, \frac{9}{16}n]$. Since $A \cup \alpha$ fixes at most $n/16$ auxiliary variables in total, their fixed values contribute at most $n/16$ to this sum, leaving a remaining target weight of roughly $n/2$. With at least $15n/16$ free y_{ik} variables available in the row, we uniformly distribute this remaining target weight across them. This results in a valid fractional assignment $\beta(y_{ik}) \in [0,1]$ (approximately $1/2$) that perfectly balances the split constraints.

4.1 Bounding the Global Column Load

Instead of bounding the capacity of each column individually, we bound the exact total load imposed on all columns by our fractional assignment to the upper rows. Because every assigned value $\beta(x_{ij})$ contributes to both a row sum and a column sum, the total column load exactly equals the sum of all satisfied upper row demands: $\sum_{j=1}^{2n+1} c_j = \sum_{i=1}^{2n+1} (n\beta(x_{ii}) + \sum_{j \neq i} \beta(x_{ij}))$.

Based on our exact assignments in the previous step, we can bound this total load:

- For the fixed rows ($i \in P_1 \cup P_0$), the assigned load depends on the diagonal. If $i \in P_0$, the load is exactly n. If $i \in P_1$, the load is $n + k_i \leq n + n/8 = \frac{9}{8}n$. In the worst case, all $n/8$ allowed fixed rows fall into P_1, meaning their maximum total contribution to the column load is $(n/8)(\frac{9}{8}n) = \frac{9}{64}n^2$.
- For the free rows ($i \in P^*$), the load is $n\beta(x_{ii}) + k_i = n\left(\frac{n-k_i}{2n}\right) + k_i = \frac{n+k_i}{2}$. Since the noise bounds guarantee $k_i \leq n/8$, each free row contributes at most $\frac{n+n/8}{2} = \frac{9}{16}n$. With at most $2n+1$ free rows, their total contribution is bounded by $(2n+1)\left(\frac{9}{16}n\right) = \frac{9}{8}n^2 + \frac{9}{16}n$.

Summing these contributions yields an upper bound on the global column load imposed by the upper matrix:

$$\sum_{j=1}^{2n+1} c_j \leq \frac{9}{64}n^2 + \frac{9}{8}n^2 + \frac{9}{16}n = \frac{81}{64}n^2 + \frac{9}{16}n \ .$$

Since the total initial capacity of any subset of columns V is $2n|V|$, the remaining available supply inside V is bounded below by the worst-case scenario where V absorbs the entirety of the global load:

$$\sum_{j \in V} S_j = 2n|V| - \sum_{j \in V} c_j \geq 2n|V| - \frac{81}{64}n^2 - \frac{9}{16}n \ . \tag{13}$$

4.2 Local Feasibility via Gale's Theorem

We must now prove that a valid fractional assignment exists for the lower rows that satisfies their demand of n without violating the remaining column capacities. We map this to a bipartite network flow problem and apply Gale's Feasibility Theorem [4].

Let U be an arbitrary subset of lower rows (demand nodes) and V be an arbitrary subset of columns (supply nodes). Gale's theorem states that a fractional assignment is feasible if and only if for all subsets U and V, the total demand of U is met by the internal supply of V plus the external connections from the remaining columns $S \setminus V$:

$$n|U| \leq \sum_{j \in V} S_j + E(U, S \setminus V) \ ,$$

where $E(U, S \setminus V)$ is the number of surviving free edges connecting U to $S \setminus V$. By the noise bounds of Theorem 4, $A \cup \alpha$ deletes at most $n/8$ edges per lower row. We verify Gale's condition across two cases based on the size of V.

4.2.0.1 Case 1 ($|V| \leq 7n/8$):

The external set $S \setminus V$ contains at least $(2n+1) - 7n/8 > 9n/8$ columns. Since every row in U loses at most $n/8$ edges to the noise $A \cup \alpha$, each row is guaranteed at least $9n/8 - n/8 = n$ valid external edges. This ensures $E(U, S \setminus V) \geq n|U|$. The feasibility condition trivially holds since $n|U| \leq 0 + n|U| \leq \sum_{j \in V} S_j + E(U, S \setminus V)$.

4.2.0.2 Case 2 ($|V| > 7n/8$):

We rearrange Gale's condition to isolate the demand forced into V: $n|U| - E(U, S \setminus V) \leq \sum_{j \in V} S_j$. The surviving external edges satisfy $E(U, S \setminus V) \geq |U|(2n + 1 - |V| - n/8)$. Thus, the left-hand demand is at most $|U|(|V| - 7n/8 - 1) \leq |U|(|V| - \frac{7}{8}n)$. Maximizing this over the $|U| \leq 2n + 2$ lower rows yields a worst-case isolated demand of $2n|V| - \frac{7}{4}n^2 + 2|V| - \frac{7}{4}n$. Combining this with the supply lower bound (13), Gale's condition holds provided:

$$2n|V| - \frac{7}{4}n^2 + 2|V| - \frac{7}{4}n \leq 2n|V| - \frac{81}{64}n^2 - \frac{9}{16}n \ .$$

Canceling $2n|V|$ and rearranging yields $2|V| \leq \frac{31}{64}n^2 + \frac{19}{16}n$. Since the total number of columns restricts $|V| \leq 2n + 1$, the left side is at most $4n + 2$. The resulting inequality $4n + 2 \leq \frac{31}{64}n^2 + \frac{19}{16}n$ is satisfied for all $n \geq 9$. Therefore, Gale's condition holds, and a valid fractional assignment exists.

We started with α that falsifies D. We extended α into β that satisfies F_n over the rationals. Because C is rationally implied, β also satisfies C. Then we round to a $0-1$ integer solution γ that still satisfies C. This contradicts our assumption that C implies D. This concludes no short clause can be derived under $F_n|_A$.

◀

4.3 The Random Restriction and Final Separation

With the width lower bound established, we can now prove the exponential size lower bound for CP+Cover by applying a random restriction.

▶ **Theorem 6** (Exponential Separation). *Any CP+Cover refutation of F_n requires length $\exp(\Omega(n))$.*

Proof. Assume for contradiction that π is a CP+Cover refutation of F_n of length $L = 2^{o(n)}$ (i.e., $L = 2^{f(n)}$ where $f(n) = o(n)$).

4.3.0.1 The Random Restriction

We define a random restriction ρ that acts only on the primary matrix variables x_{ij}, leaving all auxiliary variables y_{ik} completely free. Each variable x_{ij} is assigned independently at random as follows:

$$\Pr[\rho(x_{ij}) = *] = \frac{127}{128}, \quad \Pr[\rho(x_{ij}) = 0] = \frac{1}{256}, \quad \Pr[\rho(x_{ij}) = 1] = \frac{1}{256} \ .$$

Because ρ only assigns values to the primary variables, the survival of a clause depends strictly on its x-width. An x-clause of x-width w survives ρ (meaning no x-literal in the clause evaluates to True) with probability at most $(255/256)^w$.

4.3.0.2 High-Probability Events

We analyze two key events and show they hold simultaneously with high probability.

Event A (All clauses of large x-width are falsified): Let $C \in \pi$ be a clause of x-width $w > n/16$. Because the y-variables are left free, the probability that C survives the restriction relies entirely on its x-variables, which is at most $(255/256)^{n/16}$. By a union bound over all $L = 2^{f(n)}$ clauses in the refutation, the probability that *any* clause of large x-width survives is bounded by:

$$\Pr[A^c] \leq 2^{f(n)} \cdot \left(\frac{255}{256}\right)^{n/16} \approx 2^{f(n)} \cdot 2^{-\Theta(n)} \ .$$

Since $f(n) = o(n)$, this probability vanishes as $n \to \infty$. Thus, Event A holds asymptotically almost surely.

Event B (Restriction noise is bounded): We use standard Chernoff bounds to ensure ρ respects the noise limits required by Theorem 4.

- **Fixed diagonals**: Each diagonal x_{ii} is fixed by ρ with probability $\frac{1}{128}$. The expected number of fixed diagonals is $\mu = \frac{2n+1}{128} = \frac{n}{64} + \frac{1}{128}$. To bound the probability that this exceeds $n/16$, we note the ratio to the mean is $(1 + \delta) = \frac{n/16}{n/64+1/128} = \frac{4n}{n+0.5}$. For large n, $\delta \approx 3$. Applying the Chernoff bound yields an exponentially decaying probability $\Pr[\text{fixed diags} > n/16] \leq e^{-\Theta(n)}$.
- **Fixed variables per row/column**: A row has length $2n + 1$, so its expected number of fixed variables is $\mu_{row} = \frac{2n+1}{128} \approx \frac{n}{64}$. A column spans both the upper and lower rows for a total length of $4n+3$, meaning its expected number of fixed variables is $\mu_{col} = \frac{4n+3}{128} \approx \frac{n}{32}$. For both cases, the expectation is strictly bounded away from the threshold of $n/16$ by a constant multiplicative factor ($\delta \approx 3$ for rows, $\delta \approx 1$ for columns). Applying the Chernoff bound ensures the probability of any specific row or column exceeding $n/16$ fixed variables is $e^{-\Theta(n)}$. Taking a union bound over all $6n + 4$ rows and columns, the probability any line fails remains bounded by $(6n + 4)e^{-\Theta(n)}$.

Summing the failure probabilities for Event B yields an exponentially small quantity that vanishes as $n \to \infty$. Therefore, with positive probability, there exists a specific restriction ρ^* where both Event A and Event B hold simultaneously.

4.3.0.3 Contradiction

Consider the restricted proof $\pi\restriction_{\rho^*}$ operating on the restricted formula $F_n\restriction_{\rho^*}$.

Because Event A holds, every surviving clause in $\pi\restriction_{\rho^*}$ has width at most $n/16$. However, because Event B holds, ρ^* satisfies all the noise prerequisites of Theorem 4.

Consequently, Theorem 4 dictates that no clause of width $\leq n/16$ over the free variables can be derived. This means no matter how many linear combinations we do, on the first application of a cover cut we will always be left with a wide clause.

This contradicts the assumption that π is a valid refutation. Thus, as we know CP+Cover is sound, any refutation in CP+Cover must have length at least $L \geq \exp(\Omega(n))$. ◀

5 Conclusion

In this paper, we proved an exponential separation between cutting planes with saturation (CP+Sat) and cutting planes utilizing cover cuts (CP+Cover). By constructing the formula family F_n, we demonstrated that while saturation efficiently collapses high coefficient constraints by globally reducing coefficients, cover cuts are structurally incapable of working with such formulas efficiently.

Many open problems still remain such as:

- Does F_n give an exponential refutation separation for CP+Sat and CP+Extended Cover Cut? If using a similar restriction argument this will require some bounding of the size of the extension set E within F_n as otherwise the large cardinality constraints will be difficult to kill under the random restriction.
- This leads into the more general question of how powerful can saturation based solvers be? For example, can saturation based solvers ever be exponentially stronger than division (as posed by [3])? Does there exist further rules within LP or MIP that highlight saturations strengths?

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